Evaluate the following integrals

5B-1. $\int x\sqrt{x^2-1}\,dx$

5B-2. $\int e^{8x}\,dx$

5B-3. $\int \frac{\ln x\,dx}{x}$

5B-4. $\int \frac{\cos x\,dx}{2+3\sin x}$

5B-5. $\int \sin^2 x\cos x\,dx$

5B-6. $\int \sin 7x\,dx$

5B-7. $\int \frac{6x\,dx}{\sqrt{x^2+4}}$

5B-8. $\int \tan 4x\,dx$

5B-9. $\int e^x(1+e^x)^{-1/3}\,dx$

5B-10. $\int \sec 9x\,dx$

5B-11. $\int \sec^2 9x\,dx$

5B-12. $\int xe^{-x^2}\,dx$

5B-13. $\int \frac{x^2\,dx}{1+x^6}$. Hint: Try $u=x^3$.

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9. $\int e^x(1+e^x)^{-\frac{1}{3}}\,dx$

$u=e^x,\ du=e^x\,dx$

$\rightarrow \int (1+u)^{-\frac{1}{3}}\,du$

$= \frac{(1+u)^{\frac{2}{3}}}{2/3}+C$

$= \frac{3}{2}(1+e^x)^{\frac{2}{3}}+C$

11. $\int \sec^2 9x\,dx$

$= \frac{\tan 9x}{9}+C$

13. $\int \frac{x^2}{1+x^6}\,dx$

$u=x^3$

$\Rightarrow du = 3x^2\,dx$

$= \frac{1}{3}\int\frac{1}{1+u^2}\,du$

$\tan\theta = u$

$du = \sec^2\theta\,d\theta$

$= \frac{1}{3}\int\frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta$

$= \frac{1}{3}\int 1\,d\theta$

$= \frac{1}{3}\theta + C$

$= \frac{1}{3}\tan^{-1}u + C$

$= \frac{1}{3}\tan^{-1}x^3 + C$

Evaluate the following integrals by substitution and changing the limits of integration.

5B-14. $\int_0^{\pi/3} \sin^3 x \cos x dx$

5B-15. $\int_1^e \frac{(\ln x)^{3/2} dx}{x}$

5B-16. $\int_{-1}^1 \frac{\tan^{-1} x dx}{1+x^2}$

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16. $\int_{-1}^{1} \frac{\tan^{-1} x \, dx}{1+x^2}$

$x = \tan u$

$\Rightarrow dx = \sec^2 u \, du$

$x = 1 \Rightarrow u = \tan^{-1} 1 = \frac{\pi}{4}$

$x = -1 \Rightarrow u = \tan^{-1} -1 = -\frac{\pi}{4}$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{u \sec^2 u}{1 + \tan^2 u} du$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{u \sec^2 u}{\sec^2 u} du$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} u \, du$

$= \frac{u^2}{2} \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{1}{2} \left(\frac{\pi^2}{16} - \frac{\pi^2}{16} \right)$

$= 0$

Evaluate the following

5C-1. $\int \sin^2 x dx$ 　 5C-2. $\int \sin^3(x/2)dx$ 　 5C-3. $\int \sin^4 x dx$

5C-4. $\int \cos^3(3x)dx$ 　 5C-~~5~~. $\int \sin^3 x \cos^2 x dx$ 　 5C-6. $\int \sec^4 x dx$

5C-~~7~~. $\int \sin^2(4x)\cos^2(4x)dx$ 　 5C-8. $\int \tan^2(ax)\cos(ax)dx$ 　 5C-~~9~~. $\int \sin^3 x \sec^2 x dx$

5C-10. $\int (\tan x + \cot x)^2 dx$ 　 5C-1~~1~~. $\int \sin x \cos(2x)dx$ (Use double angle formula.)

5C-12. $\int_0^{\pi} \sin x \cos(2x)dx$ (See 27.)

5C-13. Find the length of the curve $y = \ln \sin x$ for $\pi/4 \le x \le \pi/2$.

5C-14. Find the volume of one hump of $y = \sin ax$ revolved around the x-axis.

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5. $\int \sin^3 x \cos^2 x \, dx$

$= \int (1-\cos^2 x) \sin x \cdot \cos^2 x \, dx$

$= \int (\cos^2 x - \cos^4 x) \sin x \, dx$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$\rightarrow = -\int (u^2 - u^4) \, du$

$= -\left(\frac{u^3}{3} - \frac{u^5}{5} + C_1\right)$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

7. $\int \sin^2(4x)\cos^2(4x) \, dx$

$= \int \left(\frac{1-\cos 8x}{2}\right)\left(\frac{1+\cos 8x}{2}\right) dx$

$= \frac{1}{4}\int 1 - \cos^2 8x \, dx$

$= \frac{1}{4}\int 1 - \frac{1+\cos 16x}{2} \, dx$

$= \frac{1}{4}\int \frac{1-\cos 16x}{2} \, dx$

$= \frac{1}{8}\left(x - \frac{\sin 16x}{16} + C_1\right)$

$= \frac{x}{8} - \frac{\sin 16x}{128} + C$

9. $\int \sin^3 x \sec^2 x \, dx$

$= \int \frac{\sin^3 x}{\cos^2 x} dx$

$= \int \frac{(1-\cos^2 x)\sin x}{\cos^2 x} dx$

$= \int \frac{\sin x}{\cos^2 x} - \sin x \, dx$

$= \int \frac{-du}{u^2} du - (-\cos x) + C$

$= -\frac{u^{-1}}{-1} + \cos x + C$

$= \frac{1}{\cos x} + \cos x + C$

11. $\int \sin x \cos 2x \, dx$

$= \int \sin x (1 - 2\sin^2 x) \, dx$

$= \int \sin x - 2\sin^3 x \, dx$

$= \int \sin x \, dx - 2\int (1-\cos^2 x)\sin x \, dx$

$= -\cos x + C_1 - 2\int -(1-u^2) \, du$

$= -\cos x + C_1 + 2\left(u - \frac{u^3}{3} + C_2\right)$

$= -\cos x + 2\cos x - \frac{2\cos^3 x}{3} + C$

$= \cos x - \frac{2\cos^3 x}{3} + C$

Evaluate the following integrals

5D-~~1~~. $\int \frac{dx}{(a^2-x^2)^{3/2}}$ 5D-~~2~~. $\int \frac{x^3dx}{\sqrt{a^2-x^2}}$ 5D-3. $\int \frac{(x+1)dx}{4+x^2}$

5D-4. $\int \sqrt{a^2+x^2}dx$ 5D-5. $\int \frac{\sqrt{a^2-x^2}dx}{x^2}$ 5D-6. $\int x^2\sqrt{a^2+x^2}dx$

(For 5D-4,6 use $x = a\sinh y$, and $\cosh^2 y = (\cosh(2y)+1)/2$, $\sinh 2y = 2\sinh y\cosh y$.)

5D-~~7~~. $\int \frac{\sqrt{x^2-a^2}dx}{x^2}$ 5D-8. $\int x\sqrt{x^2-9}dx$

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5D-9. Find the arclength of $y = \ln x$ for $1 \le x \le b$.

1. $\int \frac{dx}{(a^2-x^2)^{3/2}}$ $x = a\sin u \Rightarrow dx = a\cos u\, du$

$= \int \frac{a\cos u}{(a^2-a^2\sin^2 u)^{3/2}} du$

$= a\int \frac{\cos u}{a^3(1-\sin^2 u)^{3/2}} du$

$= \frac{1}{a^2}\int \frac{\cos u}{\cos^3 u} du$

$= \frac{1}{a^2}\int \sec^2 u\, du$

$= \frac{1}{a^2}(\tan u + C_1)$

$= \frac{\tan(\arcsin\frac{x}{a})}{a^2} + C$

$= \frac{x/\sqrt{a^2-x^2}}{a^2} + C$

$= \frac{x}{a^2\sqrt{a^2-x^2}} + C$

2. $\int \frac{x^3dx}{\sqrt{a^2-x^2}}$ $x = a\sin u \Rightarrow dx = a\cos u\, du$

$= \int \frac{a^3\sin^3 u \cdot a\cos u}{\sqrt{a^2-a^2\sin^2 u}} du$

$= \frac{a^4}{\sqrt{a^2}}\int \frac{\sin^3 u\cos u}{\cos u} du$

$\theta = \cos u$

$= a^3\int \sin^3 u\, du \Rightarrow d\theta = -\sin u\, du$

$= a^3\int (1-\cos^2 u)\sin u\, du$

$= a^3\int -(1-\theta^2)d\theta$

$= -a^3\left(\theta - \frac{\theta^3}{3} + C_1\right)$

$= -a^3\theta + \frac{a^3\theta^3}{3} + C$

$\theta = \cos u = \cos\left(\arcsin\frac{x}{a}\right)$

$\Rightarrow \theta = \frac{\sqrt{a^2-x^2}}{a}$

$\therefore \int \frac{x^3dx}{\sqrt{a^2-x^2}} = -a^2\sqrt{a^2-x^2} + \frac{(a^2-x^2)^{\frac{3}{2}}}{3} + C$

7. $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx$

$x = a\cos u$

$\Rightarrow dx = -a\sin u\,du$

$$= \int \frac{\sqrt{a^2\cos^2 u - a^2}}{a^2\cos^2 u}(-a\sin u\,du)$$

$$= -\frac{\sqrt{a^2}\cdot a}{a^2}\int \frac{\sqrt{\cos^2 u - 1}\cdot \sin u}{\cos^2 u}\,du$$

$$= -\int \frac{\sin^2 u}{\cos^2 u}\,du$$

$$= -\int \tan^2 u\,du$$

$$= -\int (\sec^2 u - 1)\,du$$

$$= -(\tan u + C_1) + (u + C_2)$$

$$= -\tan\left(\arccos\frac{x}{a}\right) + \arccos\frac{x}{a} + C$$

$$= -\frac{\sqrt{a^2-x^2}}{x} + \arccos\frac{x}{a} + C$$

Calculate the following integrals

5D-10. $\int \frac{dx}{(x^2+4x+13)^{3/2}}$ 5D-11. $\int x\sqrt{-8+6x-x^2}dx$ 5D-12. $\int \sqrt{-8+6x-x^2}dx$

5D-13. $\int \frac{dx}{\sqrt{2x-x^2}}$ 5D-14. $\int \frac{xdx}{\sqrt{x^2+4x+13}}$ 5D-15. $\int \frac{\sqrt{4x^2-4x+17dx}}{2x-1}$

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10. $$\int \frac{dx}{(x^2+4x+13)^{3/2}}$$

$$= \int \frac{dx}{((x+2)^2+9)^{3/2}}$$

$$= \int \frac{du}{(u^2+9)^{3/2}}$$

$$= \int \frac{3\sec^2\theta}{9^{3/2}(\tan^2\theta+1)^{3/2}}\,d\theta$$

$$= \frac{1}{9}\int \frac{\sec^2\theta}{(\sec^2\theta)^{3/2}}\,d\theta$$

$$= \frac{1}{9}\int \frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{9}\int \cos\theta\,d\theta$$

$$= \frac{1}{9}(\sin\theta + C_1)$$

$$= \frac{x+2}{9\sqrt{x^2+4x+13}} + C$$

$$x^2+4x+13$$
$$= x^2+4x+2^2-2^2+13$$
$$= (x+2)^2+9$$

$u = 3\tan\theta$

$\Rightarrow du = 3\sec^2\theta\,d\theta$

$$\sin\theta = \sin\left(\arctan\frac{u}{3}\right)$$
$$= \sin\left(\arctan\frac{x+2}{3}\right)$$
$$= \frac{x+2}{\sqrt{x^2+4x+13}}$$